



Bayesian beta regressions with brms in R: A tutorial for phoneticians

Stefano Coretta^{a,*}, Paul Bürkner^b

^a University of Edinburgh, Linguistics and English Language, 3 George Sq, Edinburgh EH8 9AD, United Kingdom

^b TU Dortmund University, Department of Statistics, Dortmund 44221, Germany

ARTICLE INFO

Article history:

Received 11 February 2025

Received in revised form 6 October 2025

Accepted 8 October 2025

Available online xxxx

Keywords:

Acoustics

Articulation

Proportions

Bayesian regressions

Beta distribution

ABSTRACT

Phonetic research frequently involves analyzing numeric continuous outcome variables, such as durations, frequencies, loudness, and ratios. Another commonly used outcome type is proportions, including measures like the proportion of voicing during closure, gesture amplitude, and nasalance. Despite their bounded nature, proportions are often modeled using Gaussian regression, largely due to the default settings of commonly used statistical functions in R (e.g., `lm()` and `lmer()` from `lme4`). This practice persists in teaching and research, despite the fact that Gaussian models assume unbounded continuous data and may poorly fit proportion data. To address this issue, this tutorial introduces beta regression models, a more appropriate statistical approach for analyzing proportions. The beta distribution provides a flexible framework for modelling continuous data constrained between 0 and 1. The tutorial employs the `brms` package in R and assumes familiarity with regression modeling but no prior knowledge of Bayesian statistics. The tutorial includes two case studies illustrating the practical implementation of Bayesian beta regression models. Data and code are available at <https://github.com/stefanocoretta/beta-phon>.

© 2025 The Author(s). Published by Elsevier Ltd. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Phonetic research often involves numeric continuous outcome variables, like durations, frequencies, loudness and ratios (Kluender, Diehl, & Wright, 1988; Johnson, 2003; Gick, Wilson, & Derrick, 2013; Ladefoged & Johnson, 2014; Abramson & Whalen, 2017; Roettger, Winter, & Baayen, 2018; Coretta et al., 2023; Rosen, 2005). Another commonly employed type of outcome variable is data that is bounded between 0 and 1: for example, proportion of voicing during closure (Davidson, 2016), vocal folds contact quotient (Herbst, Schutte, Bowling, & Svec, 2017), gesture amplitude (Carignan, 2021), nasalance (Carignan, 2021), and data from slider answers where participants can indicate responses for qualities.

Regression models (and extensions like generalised additive models) have become a *de facto* standard for the statistical analysis of a variety of measures in phonetic research (Kirby & Sonderegger, 2018; Politzer-Ahles & Piccinini, 2018; Tavakoli et al., 2024). However, there is a tendency for researchers to use Gaussian distribution families (i.e. probability distributions for the outcome variable) for any measure that is numeric, irrespective of whether the measure is unbounded,

like in truly Gaussian variables, or bounded, like in proportions. A possible reason is that the base R function for fitting regression models, `lm()`, and the `lme4` function used to fit regression models with varying terms, `lmer()` from `lme4` (Bates, Mächler, Bolker, & Walker, 2015), both fit Gaussian regressions by default and the user does not have to specify the distribution family. This tacit default of using Gaussian models is also reflected in teaching practices, where significance test and models using the Gaussian distribution are the first to be taught (Baayen, 2008; Winter, 2020), due to their relative simplicity and the fact that regression models with other families are conceptual generalisations of Gaussian regression models. Fig. 1 shows how poorly a Gaussian regression model fits beta-distributed data: the dark blue line is the density of data simulated to be distributed according to a beta distribution; the light blue lines are predicted distributions as generated by a Gaussian model of the data. The predicted Gaussian distributions over-predict values between 0.5 and 0.75 and under-predict values between 0.75 and 1. Furthermore, a Gaussian distribution predicts values above 1, which are not possible given the bounded nature of the data. Moreover, the density peak (median) of the Gaussian distributions is lower than the peak in the original data.

While most researchers approach proportions with Gaussian regression models, proportion are not Gaussian by

* Corresponding author.

E-mail addresses: s.coretta@ed.ac.uk (S. Coretta), paul.buerkner@gmail.com (P. Bürkner).

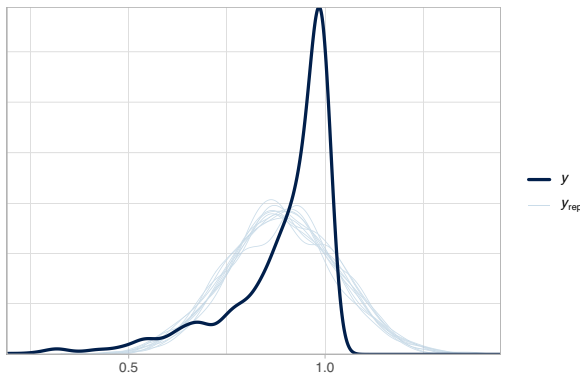


Fig. 1. Gaussian model fitted to beta-distributed data. The dark blue line is the density of the beta data, while the light blue lines are simulated densities of the same data based on a Gaussian model.

nature, since they are continuous variables bounded between 0 and 1. Thus, regression models with proportions as outcome variable should be fitted using a likelihood family that assumes such unit interval data. A common and flexible distribution for this purpose is the beta distribution. Note that the beta distribution supports variables in the interval $(0, 1)$, i.e. excluding 0 and 1; variables in the interval $[0, 1]$, including 0 and 1, can be modelled with zero and zero/one inflated beta distributions (see Section 2). This tutorial introduces researchers to beta regression models in R using the package *brms*. Familiarity with regression modelling in R with a package like *lme4* is assumed, but no prior knowledge of Bayesian statistics is necessary. The rest of the paper is structured as follows: Section 2 introduces the mathematical underpinnings of the beta distribution, Section 3 provides the readers with a brief explanation and justification of Bayesian regression models, while Section 4 and Section 5 illustrate how to fit Bayesian beta regression models with two case studies. The research compendium with data and code can be found at <https://github.com/stefanocoretta/beta-phon>.

2. The beta distribution

The beta distribution is often considered as *the* distribution for the modelling of unit interval data, i.e. data that are in the interval $[0, 1]$ (Ferrari & Cribari-Neto, 2004; Cribari-Neto & Zeileis, 2010). It has been extensively studied theoretically (Krysicki, 1999; Gupta & Nadarajah, 2014; Espinheira, Ferrari, & Cribari-Neto, 2008) and is used as a baseline to compare other unit interval distributions against (Kieschnick & McCullough, 2003; Bonat, Jr, & Zeviani, 2013; López, 2013). Specifically in Bayesian regression models, it was shown to perform well across a wide range of scenarios (Scholz & Bürkner, 2023; Scholz & Bürkner, 2025), which is why we focus on the beta distribution in this tutorial.

We use a common mean parametrisation for the beta distribution with mean parameter μ , bounded between 0 and 1, and positive precision parameter ϕ that is roughly proportional to the inverse of the variance ($\text{Var}(y) = \frac{\mu(1-\mu)}{\phi+1}$).¹ That is, the larger ϕ the smaller the variance of the corresponding beta distribution.

While the mathematical details are not needed to understand the content of this tutorial, we still write down the density below for reasons of completeness:

$$p(y|\mu, \phi) = \frac{1}{B(\mu\phi, (1-\mu)\phi)} y^{\mu\phi-1} (1-y)^{(1-\mu)\phi-1},$$

where B is the Beta function, a complex integral for whose numerical approximation efficient algorithms exist in every programming language. See Fig. 2 for an illustration of different beta distributions based on various μ and ϕ values.

As mentioned earlier, the beta distribution handles data in the interval $(0, 1)$, thus excluding 0 and 1. To model $[0, 1]$ data (including 0 and 1), the zero and zero/one inflated extensions of the beta distribution can be used. Both of these are supported in the software used in this tutorial, but will not be treated here. We refer readers to Heiss (2021). An alternative is to use the extended-support beta family (`xbeta` in *brms*). For information on the extended-support beta family, see Kosmidis and Zeileis (2024).

3. Bayesian regression models

Bayesian regression models are being increasingly adopted within phonetics and language research more broadly (Vasishth, Beckman, Nicenboim, Li, & Kong, 2018; Nalborczyk, Batailler, Loevenbruck, Vilain, & Bürkner, 2019; Verissimo, 2021). Bayesian inference involves updating of prior probability distributions in light of evidence from data, to produce posterior probability distributions. In Bayesian regression models, model parameters are modelled as full probability distributions, rather than point estimates as in Null Hypothesis Significance Testing. Given the difficulty of analytical solutions of model equations, Bayesian regressions rely on sampling algorithms to reconstruct the posterior distributions. The statistical language Stan (Stan Development Team, 2017) employs efficient and robust Markov Chain Monte Carlo algorithms for fitting a variety of models, and the R package *brms* allows R users to interface with Stan from within R to fit Bayesian regression models (Bürkner, 2017; Bürkner, 2018; Bürkner, 2021).

The main practical advantage of Bayesian regression models over maximum-likelihood-based frequentist regression models, like those fitted with the *lme4* package (Bates et al., 2015), is that Bayesian regression models don't suffer from the convergence issues that models fitted in *lme4* (Bates et al., 2015) and other packages (Cribari-Neto & Zeileis, 2010) that fit frequentist models do, independent of sample size. A second, long-term advantage is that Bayesian regression models allow researchers to statistically re-use information from previous studies by specifying informative priors. While prior specification is one of the main features of Bayesian inference, in this tutorial we will use the default priors as set by *brms*. These are sensible priors estimated from the data that facilitate convergence but bear virtually no influence on the estimated posteriors. Specifying priors requires a great deal of precise quantitative knowledge, which in some areas of phonetics is difficult to determine, so that using the default priors provided by *brms* is, for the purposes of this article (and initial analyses), an acceptable practice. For practical guidance on prior specification and prior predictive checks, see Bockting, Radev, and Bürkner (2024), Mikkola et al. (2024), Stan Development Team (2025) and McElreath (2020, p. 82-84).

¹ The μ, ϕ parametrisation is equivalent to the α, β parametrisation in that $\alpha = \mu\phi$ and $\beta = (1-\mu)\phi$. Using the former allows *brms* to exploit the same programmatic features of other distributions parametrised with a location parameter.

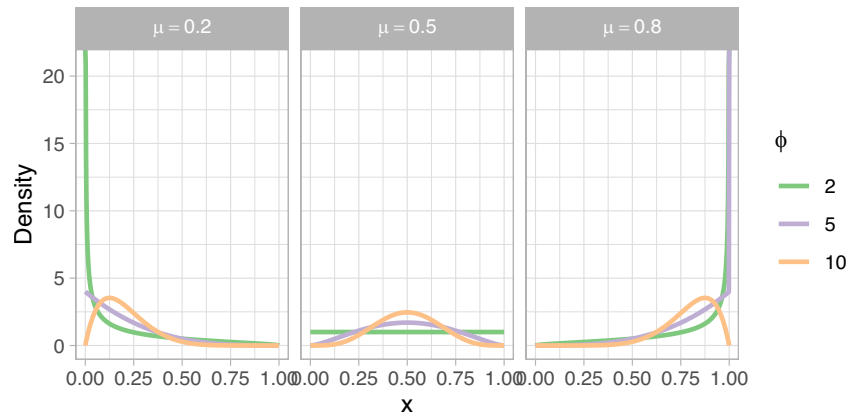


Fig. 2. Beta distributions for various μ , ϕ values.

The output of Bayesian regression models is (posterior) probability distributions for the model parameters, through which researchers can quantify (un) certainty. When obtaining the summary of the posterior probability distributions, the mean and standard deviation of the posterior distribution, together with Bayesian Credible Intervals are given for each parameter. The mean and standard deviation are commonly used metrics, given that posteriors tend to be approximately Gaussian. An alternative are the median and the median absolute deviation, which can be obtained by setting `robust = TRUE` in `summary()` (note that the nature of the posterior distribution of the parameters is not related to the nature of the distribution family chosen for the outcome, so while the outcome is modelled with a beta distribution, the posteriors of the parameters follow a symmetric distribution due to the central limit theorem). Bayesian Credible Intervals (Cris) can be calculated from the posterior distributions at several probability levels (e.g. 95, 90, 80, 60%) for a more complete view on estimated parameters. Bayesian Cris indicate that, at a certain probability level, the parameter's estimate lies within that interval: so, for example, a 90% CrI [A, B] indicates that there is a 90% probability that the estimate is between A and B. Different probability levels correspond to different levels of confidence: the higher the probability the higher the confidence (always conditional on data and model). Readers interested in a full and accessible exposition of Bayesian statistics are referred to [McElreath \(2019\)](#). Shorter introductions can be found in [Etz, Gronau, Dablander, Edelsbrunner, and Baribault \(2018\)](#), [Vasishth et al. \(2018\)](#) and [Nalborczyk et al. \(2019\)](#).

Beta regression models return coefficient estimates in log odds, similarly to Bernoulli/binomial (aka logistic) regression models. Log odds correspond to the logarithm of the odds $\log\left(\frac{p}{1-p}\right)$. To convert log odds into probabilities (or into a variable in the [0, 1] interval), the inverse logit function is used $\frac{1}{1+e^{-x}}$, where x is the log-odds value. [Fig. 3](#) illustrates the correspondence between log odds and the [0, 1] values. In R, the inverse logit can be obtained with the base R function `plogis()`, while the logit can be obtained with the base R function `qlogis()`. Another relevant transformation of log odds is exponentiation (with base R `exp()`) which returns the corresponding odds ratio: odds ratios indicate the rate of change in the values of the outcome variable relative to a

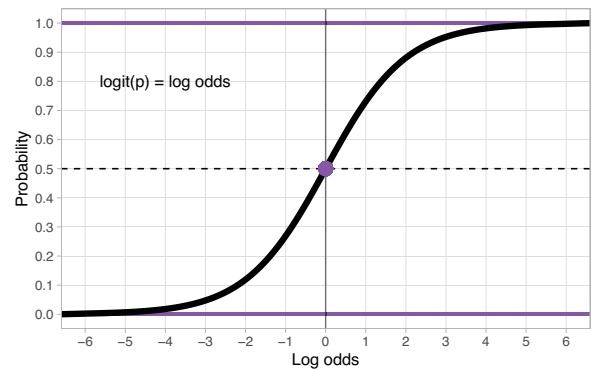


Fig. 3. Correspondence between log odds and probabilities.

baseline. An odds ratio of 1 indicates no change, positive odds ratios indicate an increase, while negative odds ratios correspond to a decrease. For example, if the odds ratio of the duration of /a/ vs the duration of /i/ is 1.2, this means that the duration of /a/ is greater than that of /i/. Moreover, odds ratios can be interpreted as percentage change: an odds ratio of 1.2 corresponds to a 20% increase relative to the baseline, an odds ratio of 0.8 corresponds to a 20% decrease relative to the baseline, while an odds ratio of 1 means a 0% change (i.e. no change).

4. Case study 1: coarticulatory voicing within consonant closure

For the first case study, we will model the proportion of coarticulatory progressive voicing within the consonant closure of voiceless stops. The measurements come from a data set of audio and electroglottographic (EGG) recordings of 19 speakers of Northwestern Italian ([Coretta, 2019; Coretta, 2020](#)). The participants read frame sentences which included target words of the form /CVCo/, where C was either /k, t, p/ in all permutations and V was either /i, e, a, o, u/ (two resulting words, /peto/ and /kako/ were excluded because they are profanities), for a total of 43 target words. There were 4 different frame sentence: *Scrivete X sul foglio* 'Write X on the sheet', *Ha detto X sei volte* 'She said X six times', *Sentivo X di nuovo* 'I heard X again', *Ripete X da sempre* 'She's been repeating X since ever'. There is a total of 172 trials per participant (3,268 grand total). The actual observation count is 2,419, after removing speech

errors, EGG measurement errors, and cases in which voicing ceased before the closure onset/after the closure offset of the second consonant in the target word.

The proportion of voicing during the closure of the second consonant was calculated as the proportion of contiguous voicing duration after closure onset to total duration of closure. The following code chunk attaches the tidyverse packages (for reading and wrangling data, Wickham et al., 2019) and loads the `ita_egg` tibble (data frame). The tibble is filtered so as to remove voicing proportions (`voi_clo_prop`) that are smaller than 0 and greater than 1. The variables `vowel` (first vowel in the target word) and `c2` (second consonant in the target word) are converted to factors to specify the order of the levels. Table 1 shows the first ten rows of the tibble (only relevant columns are included).

```
# attach tidyverse and set light theme for plots
library(tidyverse)
theme_set(theme_light())

# load tibble
load("data/coretta2018/ita_egg.rda")

# filter and mutate data
ita_egg <- ita_egg |>
  filter(voi_clo_prop > 0, voi_clo_prop < 1) |>
  mutate(
    vowel = factor(vowel, levels = c("i", "e", "a", "o", "u")),
    c2 = factor(c2, levels = c("k", "t", "p"))
  )
```

Fig. 4 shows the raw voicing duration proportion values split by vowel /i, e, a, o, u/ and second consonant /k, t, p/ in the /CVCo/ target words. The plot suggests that, on average, the voicing proportion is slightly lower with /k/ than with /t, p/. Moreover, there is greater variability between vowels in /t, p/ than in /k/. We will use a beta regression model to assess whether the place of articulation of the consonant affects the proportion of coarticulatory progressive voicing, while averaging across vowels (of course, another question that could be answered with the same model is if different vowels lead to different voicing proportions, but for sake of space we restrict this tutorial to the former question). A general expectation is that the proportion of progressive voicing should increase with fronter places of articulation (so from /k/ to /t/ to /p/) given the aerodynamic constraints which obtain with vocal fold vibration during a fully occluded oral tract (van den Berg, 1958; Rothenberg, 1967; Ohala, 2011).

We will use brms to fit a Bayesian beta regression (Bürkner, 2017). The model has voicing proportion as the outcome variable and the following terms: an interaction between vowel (/i, e, a, o, u/) and second consonant C2 (/k, t, p/), centred speech rate (number of syllables per second); as varying (aka random) terms, we included by-speaker varying coefficients for the vowel/consonant interaction and for centred speech rate.² The categorical predictors vowel and C2 are coded using index-

Table 1

First 10 rows of the data of voicing proportion within stop closure.

speaker	word	vowel	c2	voi_clo_prop
it01	poto	o	t	0.0457950
it01	topo	o	p	0.3522513
it01	pato	a	t	0.1440749
it01	teto	e	t	0.3610916
it01	toto	o	t	0.2743519
it01	puco	u	k	0.1943984
it01	chipo	i	p	0.2227896
it01	peco	e	k	0.1835596
it01	poco	o	k	0.1771007
it01	poto	o	t	0.1525163

ing rather than the traditional R contrasts (such as treatment or sum), as advocated in McElreath (2019, p 155, Ch 8): in R, this corresponds to suppressing the model's intercept with the `0 +` syntax. With this approach, the model's coefficients do not represent differences from a reference level (normally the intercept), rather they represent the predicted mean for all combinations of each categorical predictor's levels (no contrasts are applied). The following code shows the coding matrix for a hypothetical predictor with three levels, coded using treatment contrasts and indexing.

```
fac <- factor(c("a", "b", "c"))
```

```
# Treatment contrasts
contrasts(fac)
```

```
  b  c
a  0  0
b  1  0
c  0  1
```

```
# Indexing
diag(nlevels(fac))
```

```
  [,1] [,2] [,3]
[1,]  1   0   0
[2,]  0   1   0
[3,]  0   0   1
```

With indexing, a model would have one coefficient for the mean of "a", one for the mean of "b", and one for the mean of "c". No comparisons between levels are present in the matrix, but these can be obtained from the model post hoc (as we do below).

One benefit of using indexing is that uncertainty is equally distributed across all levels of categorical predictors, since priors apply on the mean of each level, rather than on the difference of subsequent levels from the reference level. The latter scenario, which applies in traditional contrasts, including treatment and sum, makes estimates of the last level in the predictor more uncertain than the estimates of the other levels, given that the uncertainty of the intercept is added to the uncertainty of the difference. As explained in Section 3, using the default priors calculated by brms is an acceptable method in the absence of very specific quantitative expectation, given that

² While the terms "random effects, intercept and slopes" are commonly employed in applied statistical work in linguistics and phonetics, we opt to use the conceptually more appropriate terminology proposed by Gelman (2005): varying terms, intercepts, slopes, coefficients. Other common terms are multilevel and hierarchical terms. Note that with "varying terms" we refer specifically to model terms with partial pooling in opposition to "constant terms" estimated with no pooling (McElreath, 2020, p. 408). See Nalborczyk et al. (2019) for a detailed overview.

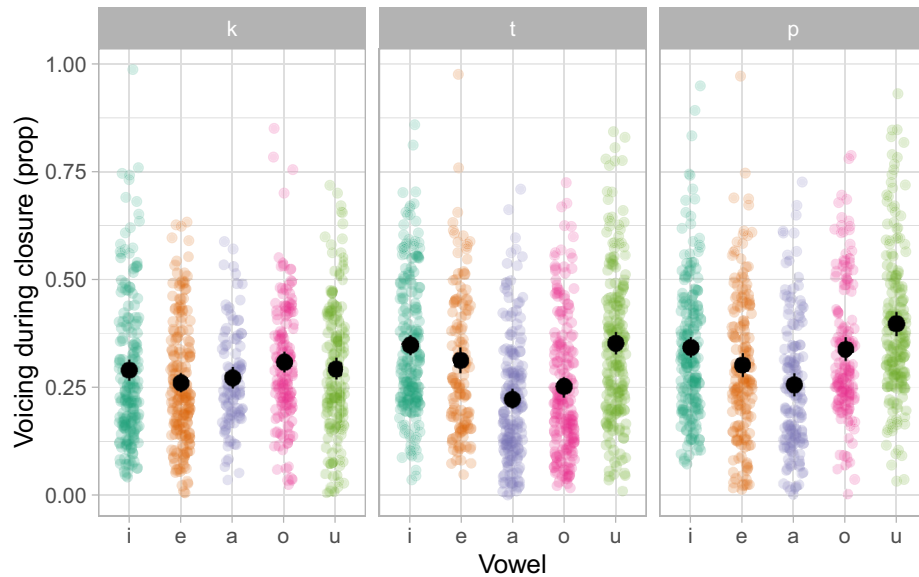


Fig. 4. Proportion of voicing during the closure of the second consonant in /CVC/ words by vowel and the second consonant.

the default priors are weak enough to bear little effect on posteriors. Researchers should use their own judgement and expertise in a case by case fashion.

```
# attach brms
library(brms)

# fit the model
# Takes 3 minutes on MacBook Pro 2020, M1
voi_prop_bm <- brm(
  # model formula
  voi_clo_prop ~
    # constant terms
    0 + vowel:c2 + speech_rate_c +
    # varying terms
    (0 + vowel:c2 + speech_rate_c | speaker),
  # uses the beta family for the outcome
  family = Beta,
  data = ita_egg,
  cores = 4,
  seed = 3749,
  file = "data/cache/voi_prop_bm"
)
```

The `summary()` function prints the full model summary. For conciseness, we will use the `fixef()` function which prints the regression coefficients. Table 2 reports the output of `fixef()` as a table (we round all values to the nearest 2 digits for clarity). For each coefficient in the model, `fixef()` prints the name of the coefficient, the mean estimate (i.e. the mean of the posterior distribution calculated from the MCMC draws), the estimate error (i.e. the standard deviation of the posterior distribution) and the lower and upper limits of a Bayesian Credible interval (CrI) of the posterior distribution. Note that all values are in log odds, since the link function in beta regressions is the logistic function, like in binomial/Bernoulli regressions. Here, we print an 80% CrI. There is nothing special about 95% CrI within Bayesian inference and instead experts recommend to check and report a variety of CrIs. Obtaining CrIs at

Table 2

Regression coefficients of a beta regression of voicing proportion (`voi_prop_bm`).

	Estimate	Est.Error	Q10	Q90
speech_rate_c	0.08	0.06	0.01	0.15
voweli:c2k	-0.91	0.14	-1.08	-0.74
vowele:c2k	-1.08	0.11	-1.22	-0.94
vowela:c2k	-0.99	0.12	-1.14	-0.84
vowelo:c2k	-0.79	0.14	-0.96	-0.62
vowelu:c2k	-1.00	0.16	-1.20	-0.80
voweli:c2t	-0.66	0.11	-0.79	-0.53
vowele:c2t	-0.84	0.14	-1.02	-0.66
vowela:c2t	-1.43	0.13	-1.60	-1.26
vowelo:c2t	-1.15	0.13	-1.31	-0.99
vowelu:c2t	-0.68	0.12	-0.83	-0.54
voweli:c2p	-0.68	0.11	-0.81	-0.54
vowele:c2p	-0.88	0.15	-1.07	-0.68
vowela:c2p	-1.14	0.13	-1.31	-0.98
vowelo:c2p	-0.66	0.11	-0.80	-0.53
vowelu:c2p	-0.44	0.12	-0.59	-0.28

different probability levels allows researchers to make more fine-grained inferential statements than the frequentist significance dichotomy affords. For simplicity of exposition, we will use 80% CrIs in this case study but we strongly recommend researchers to always obtain CrIs at different levels of probability and base their inferences on all and not one in particular. To reiterate, in Bayesian inference, an 80% CrI indicates the range of values within which the true estimate falls at 80% probability or confidence.

```
fixef(voi_prop_bm, prob = c(0.1, 0.9))
```

From the summary, we see that speech rate (number of syllables per second) has a positive effect on voicing proportion: the 80% CrI is between 0.01 and 0.15 log odds [$\beta = 0.08$, $SD = 0.06$]. Log odds can be converted to odds ratios by exponentiating the value: 0.01–0.15 log odds correspond to an odds ratio of 1.01 to 1.16, or as percentages, to an increase of voicing of 1 to 16% for every increase of one syllable per second.

Since this is an 80% CrI, we can be 80% confident that the true effect of speech rate is between 1–16% increase of voicing proportion, conditional on the data and model. Note that transforming measures this way is appropriate *only* with quantile-based measures (like CrIs) but not with moments like the mean and standard deviation: to correctly get mean and SDs in the transformed scale, you must first extract the posterior draws (see below), convert them and then take moments such as mean and SD (for a more detailed explanation, see Bürkner, 2025, p. 101). In the avoidance of doubt, we will always transform the drawn values first and then take summary measures.

Turning now to the coefficients for vowel and C2, given the indexing approach of coding these variables the model summary and the output of `fixef()` reports the *predictions* in log odds for each combination of vowel and C2, rather than differences between levels. The posteriors of the coefficients for vowel and C2 are the estimated μ s of the beta distribution, since the beta distribution is parametrised with μ and ϕ . We will not discuss ϕ , although the estimate of this parameter can be found in the summary of the model, under “Further Distributional Parameters”. The CrIs of the vowel /C2 coefficients span all negative log-odds values: these correspond to proportions that are lower than 0.5 (which is 0 in log odds). This matches the general trends in the raw data, which we plotted in Fig. 4. Next, we will plot the predicted proportions of each vowel /C2 combination at mean speech rate (i.e. centred speech rate = 0) and then calculate the average pair-wise difference in voicing proportion between /k, t, p/.

Before being able to plot the predictions, it’s important to get familiar with the so-called posterior draws. Bayesian regression models are fitted using Markov Chain Monte Carlo (MCMC) methods, as briefly explained in Section 3. The `brm()` function runs four MCMC chains with 2000 iterations each by default. These are sufficient in most cases. The first 1000 iterations of each chain are used for “warmup”, where the MCMC algorithm is optimised to find the posterior probability. The second half of the iterations, 1000 per chain, are used for sampling from the posterior probability. Each iteration returns a drawn value for each parameter in the model. The list of values drawn during the Monte Carlo sampling are called the “posterior draws”. Since four chains are run, a total of 4000 posterior values are generated. The posterior draws are used for inference: we can plot them, summarise them, transform them and combine them in any way to answer specific questions.

Posterior draws can be conveniently obtained with the `as_draws_df()` from `brms`. For the moment, we will extract only the draws of the constant (aka fixed) regression coefficients (model variables starting with `b_`). To check which coefficients are available in a model, use `get_variables()` from the `tidybayes` package (Kay, 2019). `as_draws_df()` returns a tibble where each column contains the drawn values of a coefficient. Table 3 shows the first ten rows and first five columns of the output of `as_draws_df()`. The probability distribution of the drawn values of each coefficient is the posterior probability distribution of that coefficient. Note that, due to the indexing coding of vowel and C2, all coefficient except `b_speech_rate_c` are drawn *log-odds values* for each vowel/C2 combination (the drawn values for `b_speech_rate_c` are drawn *differences* in log-odds values for each unit increase of speech rate). The drawn values are in log odds, but we can convert them to proportions with `plogis()` (we will do this when plotting below).

```
# extract only coefficient variables starting with "b_"
voi_prop_bm_draws <- as_draws_df(voi_prop_bm, variable =
  "b_", regex = TRUE)
```

We can now wrangle this tibble and plot the posterior distributions for each vowel/C2 combination. Table 4 shows the first ten rows.

```
voi_prop_bm_draws_long <- voi_prop_bm_draws |>
# drop b_speech_rate_c before pivoting
select(-b_speech_rate_c) |>
# pivot vowel:c2 columns
pivot_longer(`b_vowel:c2k`:`b_vowel:c2p`, names_to =
  "coeff") |>
# separate "coeff" labels into type ("b"), vowel and c2
separate(coeff, into = c("type", "vowel", "c2"))
```

For plotting, we can use `ggplot2` statistics layers from the `ggdist` package [Kay (2023); kay2024]. `stat_halfeye()` plots the density of the posterior probability (in grey), its median (point) and CrIs (lines). Let’s use 60 and 80% CrIs and transform the log-odds values to proportions with `plogis()`. See Fig. 5.

Table 3
First ten rows and 5 columns of the posterior draws for the model `voi_prop_bm`.

<code>b_speech_rate_c</code>	<code>b_vowele:c2k</code>	<code>b_vowelo:c2k</code>	<code>b_vowele:c2p</code>	<code>b_vowelo:c2p</code>
0.1593890	-0.7213516	-1.046577	-1.1197185	-1.0191525
0.0351033	-0.7492306	-1.385450	-1.0633164	-0.8854320
0.1259352	-0.8472495	-0.975776	-1.0972735	-0.6440836
0.0640492	-0.7033469	-1.068626	-1.0475190	-0.8328252
0.0319081	-0.7659281	-1.177656	-0.9357499	-0.6955130
0.0729965	-0.8494392	-1.226692	-1.0635364	-0.7692243
0.1009609	-0.9546148	-1.072955	-1.1408255	-0.8566750
0.0735392	-1.0160675	-1.065025	-1.1736263	-0.8334884
0.0870061	-1.0436475	-1.069337	-1.2632061	-0.7941857
0.0923815	-1.2042168	-1.041238	-0.9158689	-0.6444492

```
# attach ggdist package
library(ggdist)

voi_prop_bm_draws_long |>
  mutate(
    c2 = str_remove(c2, "c2") |> factor(levels = c("k", "t", "p")),
    vowel = str_remove(vowel, "vowel")
  ) |>
  ggplot(aes(plogis(value), vowel)) +
  stat_halfeye(.width = c(0.6, 0.8)) +
  facet_grid(rows = vars(c2)) +
  labs(
    caption = "The points are the medians and the two lines are
    60 and 80% CrIs.",
    x = "Voicing during closure (prop)",
    y = "Vowel"
  )
```

What if we want to plot the average predicted voicing proportion for the three consonants /k, t, p/? We can “marginalise” over vowel, i.e. take the mean across vowels within each consonant for each posterior draw, and the posterior distribution of the resulting list of values is the predicted posterior distribution of voicing proportion for each consonant, assuming an “average” vowel. See Fig. 6 for the density of these posteriors. For a detailed treatment of various approaches to marginalisation that apply to more complex cases like those involving numeric predictors, see Arel-Bundock, Greifer, and Heiss (2024).

```
voi_prop_bm_draws_long_c2 <- voi_prop_bm_draws_long |>
  # grouping by .draw and c2 ensures that averaging applies only
  # within draw and c2
  group_by(.draw, c2) |>
  # calculate the mean value within draw/c2
  summarise(
    value_mean = mean(value), .groups = "drop"
  )
```

```
voi_prop_bm_draws_long_c2 |>
  mutate(
    c2 = str_remove(c2, "c2") |> factor(levels = c("k", "t", "p"))
  ) |>
  ggplot(aes(plogis(value_mean), c2)) +
  stat_halfeye(.width = c(0.6, 0.8)) +
  labs(
    caption = "The points are the medians and the two lines are 60
    and 80% CrIs.",
```

```
x = "Voicing during closure (prop)",
y = "C2"
)
```

Based on the expected posterior distributions of the mean voicing proportion by consonant, /p/ has a somewhat higher voicing proportion than /k/ and /t/. The real question is: how much higher? We can quantify this by taking the difference of the drawn values for /p/ and those for /k, t/ (all averaged across vowels). Since we want to compare /k, t/ with /p/, we should first average the average draws of /k, t/ and then take the difference of the averaged /k, t/ draws and the draws of /p/. Table 5 shows the first ten rows of the resulting data frame. The posterior distribution of the expected difference is shown in Fig. 7.

```
voi_prop_bm_diff <- voi_prop_bm_draws_long_c2 |>
  # pivot data to create one column per consonant with the mean
  # drawn values,
  # with one draw per row
  pivot_wider(names_from = c2, values_from = value_mean) |>
  mutate(
    # calculate the mean of /k/ and /t/, for each draw
    c2tk = mean(c(c2k, c2t)),
    # calculate the difference of /p/ and /k, t/
    c2p_tk_diff = c2p - c2tk
  )
```

```
voi_prop_bm_diff |>
  ggplot(aes(c2p_tk_diff)) +
  stat_halfeye(.width = c(0.6, 0.8, 0.9)) +
  geom_vline(xintercept = 0) +
  labs(
    x = "Log-odds difference in /p/ vs /k, t/",
    y = element_blank()
  )
```

Once we have the posterior difference, we can obtain CrIs of the difference using `quantile2()` from the posterior package (Bürkner, Gabry, Kay, & Vehtari, 2024). Beware that the values of the difference are in log odds! We can convert these into odds ratios with `exp()`. The output of the code below is shown in Table 6. Odds ratios indicate the ratio of the difference between A and B, so that 1 means no difference, values greater than 1 indicate an increase in A relative to B and values lower than 1 indicate a decrease in A relative to B. Odds ratios are useful

Table 4

First ten rows of posterior draws from `voi_prop_bm` in long format.

.chain	.iteration	.draw	type	vowel	c2	value
1	1	1	b	voweli	c2k	−0.7213516
1	1	1	b	voweie	c2k	−1.0465766
1	1	1	b	vowela	c2k	−1.1197185
1	1	1	b	vowelo	c2k	−1.0191525
1	1	1	b	vowelu	c2k	−1.0523007
1	1	1	b	voweli	c2t	−0.6248875
1	1	1	b	voweie	c2t	−0.7951552
1	1	1	b	vowela	c2t	−1.3999773
1	1	1	b	vowelo	c2t	−1.0950180
1	1	1	b	vowelu	c2t	−0.6682030

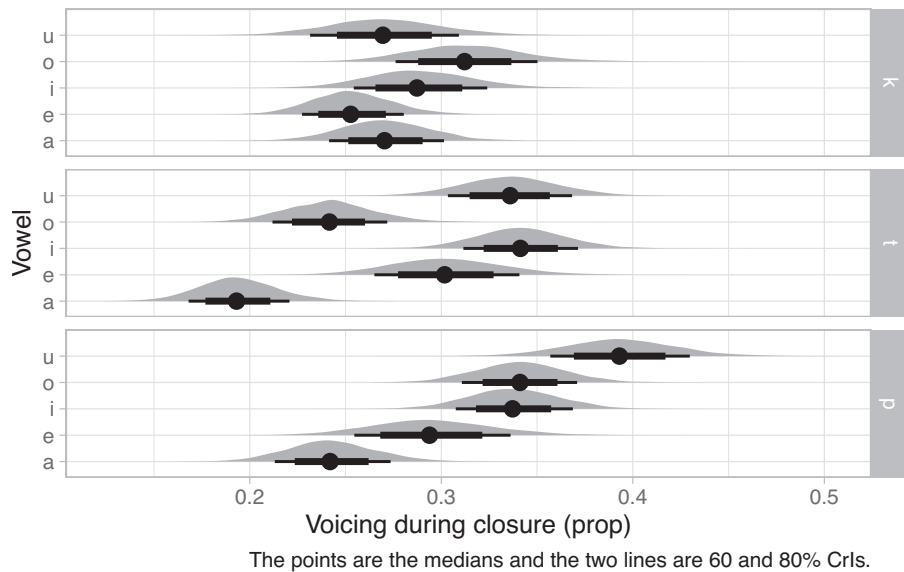


Fig. 5. Expected predictions of voicing proportion by vowel and consonant from `voi_prop_bm`.

when looking at differences that are in log odds because while the relative magnitude of the difference in proportion between two groups is the same independent of the baseline proportion, the *absolute* magnitude of the difference depends on the baseline value. For example, an odds ratio difference of 1.25 would correspond to a proportion increase of about 3 percentage points if the baseline proportion is 0.62 but it would correspond to a proportion increase of 5 percentage points if the baseline proportion is 0.73 and 1 percentage point if the baseline is 0.99. Of course, in real research contexts it is still useful to think about absolute magnitudes and their relevance from a conceptual and methodological perspective. In this tutorial we just focus on odds ratios for simplicity.

```
library(posterior)

voi_bm_quant <- voi_prop_bm_diff |>
  mutate(c2p_tk_diff_ratio = exp(c2p_tk_diff)) |>
  reframe(
    # 90% CrI
    q90 = quantile2(c2p_tk_diff_ratio, probs = c(0.05, 0.95)),
    # 80% CrI
    q80 = quantile2(c2p_tk_diff_ratio, probs = c(0.1, 0.9)),
    # 60% CrI
    q60 = quantile2(c2p_tk_diff_ratio, probs = c(0.2, 0.8)),
  ) |>
  # round to 2 digits
  mutate(across(everything(), round(.x, 2)))
```

Based on the model and data, there is a 90% probability that the voicing proportion in /p/ is 1.07–1.38 times longer (or 7–38% increase) than in /k, t/. At 80% confidence, the change ratio is 1.10–1.34 (or 10–34% increase) while at 60% confidence is 1.14–1.30 (14–30% increase). In other words we can be quite confident that the voicing proportion in /p/ is greater than in /k, t/ and that the increase is less than 35%. The `brms` package comes with a convenient function, `conditional_effects()`, to plot posterior means and CrIs based on predictors in the model. In Fig. 8, we plot the predicted proportion of voicing by consonant and vowel.

```
ce <- conditional_effects(voi_prop_bm, "c2:vowel")
p <- plot(ce, plot = FALSE)[[1]]
p + labs(x = "C2", y = "Voicing during closure (prop)")
```

Finally, the package `marginalEffects` (Arel-Bundock, Greifer, & Heiss, 2025) has two other convenience functions that return CrIs of comparisons across predictor levels (`avg_comparisons()`) and CrIs of posterior predictions across predictor levels (`avg_predictions()`). Both functions take the model and a list of variables as arguments. Optionally, you can set the confidence level with the `conf_level` argument (here, we set it to 0.8, or 80%). `avg_comparisons()` allows the user to specify the type of comparison for each variable: here we set "pairwise" to calculate pairwise comparisons of each level of `c2`. The output of the two functions is shown in Table 7 and Table 8.³

```
library(marginalEffects)

avg_comparisons(voi_prop_bm, variables = list(c2 = "pairwise"),
  conf_level = 0.8, type = "link")
avg_predictions(voi_prop_bm, variables = "vowel", conf_level =
  0.8)
```

In sum, we used a Bayesian beta regression model to assess whether different places of articulation affect the proportion of coarticulatory progressive voicing. The expectation was that the proportion of progressive voicing should increase with fronter places of articulation (so from /k/ to /t/ to /p/). This expectation is not borne out by the results: while the proportion of voicing is robustly greater in /p/ than in /k, t/ (at 90%

³ For a discussion on multiple comparisons in Bayesian statistics, see Gelman, Hill, and Yajima (2012). If multiple comparisons fixed effects is performed and one worries about their joint frequentist error rates, one could, for example, increase the width of the credible intervals for more conservative inference.

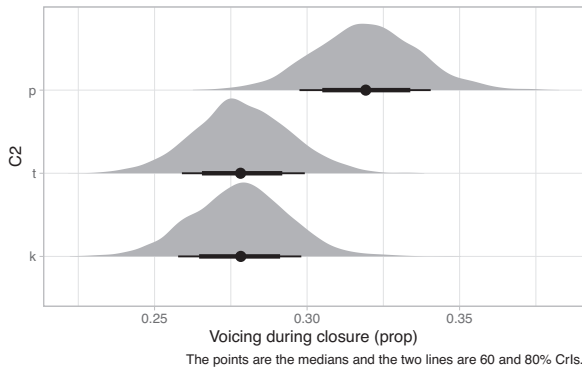


Fig. 6. Expected predictions of voicing proportion by consonant, averaged across vowels. The points are the medians and the two lines are 60 and 80% CrIs.

probability, the voicing proportion in /p/ is 7–38% higher), the difference in voicing proportion between /k/ and /t/ is very small and uncertain.

5. Case study 2: coarticulatory vowel nasalisation

For the second case study we will use data from Carignan (2021). The study looked at properties of nasality in German VNC sequences. Here, we will focus on the effect of C voicing (voiceless /t/ vs voiced /d/) on the proportion of nasalisation within the vowel in the VNC sequence. Previous work on coarticulatory nasalisation in English has suggested that vowels followed by an NC sequence where C is voiceless (NT) should show earlier coarticulatory nasalisation than vowels followed by an NC sequence where C is voiced (ND). This pattern has been suggested to be driven by the articulatory and acoustic incompatibility of voicelessness and nasalisation, by which the velum opening gesture of the nasal consonant is pushed away (i.e. earlier) when the consonant following the nasal is voiceless (Fowler, 2005; Beddor, 2009; Cho, Kim, & Kim, 2017; Carignan, Shosted, Fu, Liang, & Sutton, 2015). Everything else being equal, a greater proportion of vowel nasalisation (from the perspective of time) should be found in vowels followed by NT than in vowels followed by ND.

We will model the proportion of coarticulatory nasalisation in the German short vowels /i, e, a, o, u/ when followed by /nt/ or /nd/, using a Bayesian beta regression model. The proportion was calculated as the proportion of the nasal interval to the duration of the vowel. The nasal interval was defined thus: the interval between the time of peak velocity of velum opening to the offset of the vowel. We will use the results of the regression model to answer the following questions:

Table 5

First 10 rows of expected difference of voicing proportion between /k, t/ and /p/.

.draw	c2k	c2p	c2t	c2tk	c2p_tk_diff
1	−0.9918200	−0.7450832	−0.9166482	−0.9533426	0.2082595
2	−1.0240673	−0.7904348	−1.0322325	−0.9533426	0.1629079
3	−0.8954946	−0.6873228	−0.8904412	−0.9533426	0.2660199
4	−0.9442361	−0.8135191	−0.9095122	−0.9533426	0.1398235
5	−0.9278262	−0.7908457	−0.9200903	−0.9533426	0.1624970
6	−0.9825895	−0.7283054	−0.9262196	−0.9533426	0.2250372
7	−1.0482359	−0.8731592	−1.0841208	−0.9533426	0.0801835
8	−1.0613713	−0.8751219	−1.0741555	−0.9533426	0.0782208
9	−1.0622522	−0.9739388	−1.0481834	−0.9533426	−0.0205962
10	−0.9293998	−0.8287910	−0.9526105	−0.9533426	0.1245517

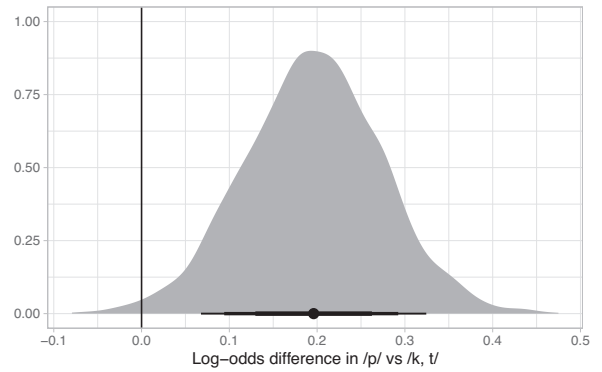


Fig. 7. Posterior distribution of the difference between the expected voicing proportion of /k, t/ vs /p/, averaged across vowels.

Table 6

Upper and lower limits of 90, 80 and 60% Credible Intervals of the difference ratio of voicing proportion in /k, t/ vs /p/.

q90	q80	q60
1.07	1.10	1.14
1.38	1.34	1.30

1. Is the nasalisation proportion, on average across vowels, greater in voiceless NC sequences?
2. Is there individual speaker variation?

Table 9 shows the first ten rows of the data. The data contains the following columns:

- `speaker` indicates the speaker ID.
- `label` is the word label as given in the original data.
- `vowel` is the target vowel in the VNC sequence.
- `NC` is the NC sequence (voiceless /nt/ or voiced /nd/).
- `voicing` indicates the voicing of C.
- `nas_prop` is the proportion of coarticulatory nasalisation of the vowel.

```
nasal <- read_csv("data/carignan2021/nasal.csv")
```

Fig. 9 shows the proportion of coarticulatory nasalisation in vowels followed by /nd/ (voiced) vs /nt/ (voiceless) sequences, for the short vowels /i, e, a, o, u/. We can see a pattern of

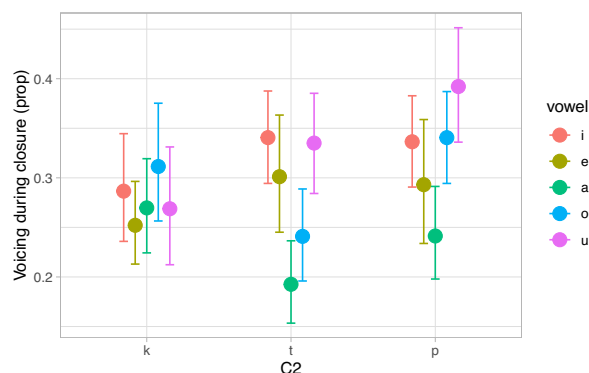


Fig. 8. Expected voicing proportion by consonant and vowel with 95% Credible Intervals.

Table 7

Difference in expected voicing proportion by consonant, averaged across vowels, with 80% Crls.

term	contrast	estimate	conf.low	conf.high
c2	p - k	0.218	0.178	0.256
c2	p - t	0.183	0.145	0.219
c2	t - k	0.035	-0.004	0.074

Table 8

Expected voicing proportion by vowel, averaged across vowels, with 80% Crls.

vowel	estimate	conf.low	conf.high
i	0.331	0.325	0.338
e	0.300	0.293	0.307
a	0.245	0.238	0.252
o	0.305	0.298	0.312
u	0.348	0.341	0.354

higher nasalisation proportion in vowels followed by /nt/, at least in the vowels /a, i, o/. For /e, u/, the distribution of nasalisation proportion seems to be similar between the voiced and voiceless contexts.

Now onto modelling with a beta regression. Note that a full appropriate model would include further predictors (both constant and varying), but for simplicity here we include only the following predictors: voicing (voiced /nd/ vs voiceless /nt/), vowel (/i, e, a, o, u/), including an interaction between them. As varying terms, we include a varying intercept by speaker and a by-speaker varying slope for voicing and vowel in interaction. As with the model from the first case study, voicing and vowel are coded using indexing, by suppressing the intercept with 0 +. Here's the code of the model.

```
nas_prop_bm <- brm(
  nas_prop_0 + voicing:vowel + (0 + voicing:vowel ~ speaker),
  data = nasal,
  family = Beta,
  cores = 4,
  seed = 3749,
  file = "data/cache/nas_prop_bm"
)
```

Let's inspect the output of `fixef()`, reported in Table 10.

```
fixef(nas_prop_bm, prob = c(0.1, 0.9))
```

Negative log odds indicate a proportion that is smaller than 50%, while positive log odds indicate a proportion that is greater than 50%. Generally, the expected log-odds predictions in Table 10 are negative, indicating an overall tendency for the nasalisation to take less than 50% of the duration of the vowel. Moreover, the predictions are higher for voiceless NC sequences than for voiced NC sequences, indicating a greater proportion of nasalisation in the former. However there is vowel-specific variation, and there doesn't seem to be much of a difference in nasalisation proportion in /e/ and /u/. Fig. 10 shows the expected predictions with `conditional_effects()`, which should make the interpretation of the predictions from Table 10 clearer.

```
ce <- conditional_effects(nas_prop_bm, "vowel:voicing")
p <- plot(ce, plot = FALSE)[[1]]
p +
  labs(
    y = "Proportion of nasality"
  )
```

Now that we fitted the model we can use the posterior draws to answer the two research questions (repeated from above):

1. Is the nasalisation proportion, on average across vowels, greater in voiceless NC sequences?
2. Is there individual speaker variation?

To answer question 1, we can calculate the average difference in nasalisation proportion by first calculating the average nasalisation across all vowels for voiced and voiceless sequences (see Table 11 for the output of this step) and then take the difference of those, similarly to what we have done in the case study in Section 4.

```
# extract only coefficient variables starting with "b_"
nas_prop_bm_draws <- as_draws_df(nas_prop_bm, variable =
  "b_", regex = TRUE)

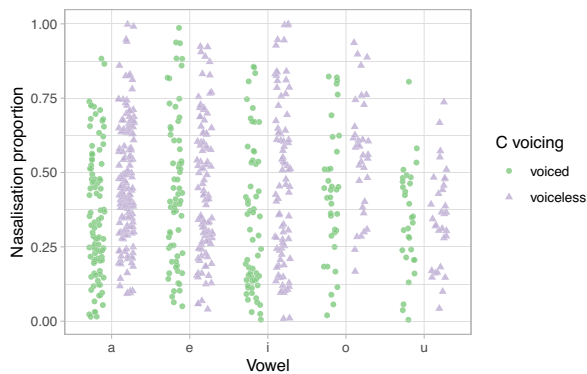
nas_prop_bm_draws_long <- nas_prop_bm_draws |>
  # pivot vowel:c2 columns
  pivot_longer("b_voicingvoiced:vowela":"b_voicingvoiceless:vo
  welu", names_to = "coeff") |>
  # separate "coeff" labels into type ("b"), vowel and c2
  separate(coeff, into = c("type", "vowel", "c2"))
```

Now let's calculate the mean nasalisation proportion within each draw by voicing, and plot the resulting posterior distributions. Note that, as discussed in Section 4, when working with log odds it is important to first do all necessary calculations in log-odds (here calculate the mean log odds across vowels) and then transform the calculated estimands to proportions/

Table 9

First ten rows of the nasal proportion data.

speaker	label	vowel	NC	voicing	nas_prop
S03	b_U_nt@_N_B17/s	u	nt	voiceless	0.3668820
S03	b_a_nd@_N_B19/s	a	nd	voiced	0.1954858
S03	b_a_nt@_N_B15/s	a	nt	voiceless	0.2786485
S03	f_I_nt@_N_B05/s	i	nt	voiceless	0.7642259
S03	l_I_nd@_N_B06/s	i	nd	voiced	0.0052949
S03	p_E_nt@_N_B09/s	e	nt	voiceless	0.3347331
S03	r_a_nt@_N_B06/s	a	nt	voiceless	0.2431760
S03	v_I_nd@_N_B07/s	i	nd	voiced	0.0247572
S03	v_I_nt_6_N_B15/s	i	nt	voiceless	0.1350081
S03	z_E_nd@_N_B17/s	e	nd	voiced	0.5378522

**Fig. 9.** Proportion of coarticulatory nasalisation during the vowel in VNC sequences in German, depending on C voicing.

probabilities. The output of the following code is shown in Table 12 and the density plot of the calculated draws is in Fig. 11.

```
nas_prop_bm_draws_long_voicing <- nas_prop_bm_draws_long
|>
# grouping by .draw and voicing ensures that averaging applies
# only within draw and voicing
group_by(.draw, voicing) |>
summarise(
  # calculate the mean value within draw/voicing in log odds
  value_mean = mean(value),
  # we can now transform log odds to proportion with plogis()
  value_mean_prop = plogis(value_mean),
  .groups = "drop"
)
```

```
nas_prop_bm_draws_long_voicing |>
mutate(
  voicing = str_remove(voicing, "voicing")
) |>
ggplot(aes(value_mean_prop, voicing)) +
stat_halfeye(.width = c(0.6, 0.8)) +
labs(
  x = "Nasalisation proportion"
)
```

The plot suggests an overall greater nasalisation proportion in voiceless NC sequences. Let's quantify how much greater it is, as we did in Section 4. We will use odds ratios in this con-

text as well, i.e. we will convert log odds to odds ratios using the `exp()` (exponential) function (and as before we first calculate the difference and then exponentiate the resulting values, after which we can take summary measures, like means and quantile-based measures such as CrIs). Table 13 show the 90, 80 and 60% CrIs of the difference ratio of nasalisation proportion in voiceless vs voiced NC sequences.

```
nas_prop_bm_diff <- nas_prop_bm_draws_long_voicing |>
# pivot data to create one column per voicing with the mean
# drawn values,
# with one draw per row. we need to drop the value_mean_prop
# col
select(-value_mean_prop) |>
pivot_wider(names_from = voicing, values_from =
  value_mean) |>
mutate(
  # calculate the difference of voiceless and voiced in log odds
  voicing_diff = voicingvoiceless - voicingvoiced,
  # now transform with exp() to get the ratio difference
  voicing_diff_ratio = exp(voicing_diff)
)

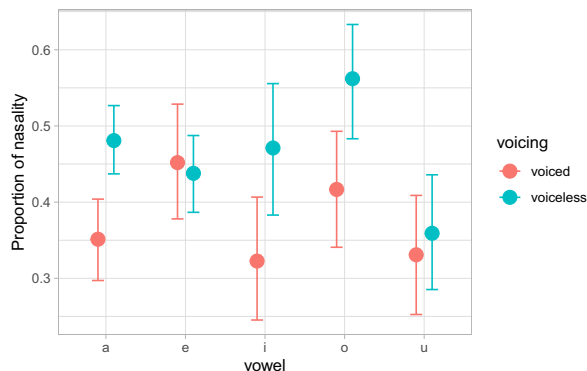
nas_prop_bm_diff_quant <- nas_prop_bm_diff |>
reframe(
  # 90% CrI
  q90 = quantile2(voicing_diff_ratio, probs = c(0.05, 0.95)),
  # 80% CrI
  q80 = quantile2(voicing_diff_ratio, probs = c(0.1, 0.9)),
  # 60% CrI
  q60 = quantile2(voicing_diff_ratio, probs = c(0.2, 0.8)),
) |>
mutate(across(everything(), round(.x, 2)))
```

The CrIs of the ratio difference in nasalisation proportion in voiceless vs voiced NC sequences suggest an increase of nasalisation in the voiceless NC sequences, with a 90% probability that the increase is between 23% and 69% of the proportion in voiced NC sequences.

Moving onto question 2: is there individual speaker variation? While in the recent tradition of linguistic research, varying (aka random) effects are used to *control* for differences across participants, it has been proposed to use the estimates of the varying coefficients to investigate individual differences (Tamminga, Ahern, & Ecay, 2016). We will use the `spread_draws()` function from `tidybayes` (Kay, 2019) to extract the draws of the varying terms (in brms these are the coefficients that start with `r_`). There is quite a few steps of processing to get from the raw draws to the estimand we need: while we

Table 10Regression coefficients of a beta regression of nasalisation proportion (*nas_prop_bm*).

	Estimate	Est.Error	Q10	Q90
voicingvoiced:vowela	−0.61	0.12	−0.77	−0.46
voicingvoiceless:vowela	−0.08	0.09	−0.19	0.04
voicingvoiced:vowe	−0.19	0.16	−0.38	0.01
voicingvoiceless:vowe	−0.25	0.10	−0.38	−0.12
voicingvoiced:vowel	−0.74	0.19	−0.98	−0.51
voicingvoiceless:vowel	−0.12	0.18	−0.35	0.10
voicingvoiced:vowel	−0.34	0.16	−0.54	−0.14
voicingvoiceless:vowel	0.25	0.15	0.05	0.44
voicingvoiced:vowel	−0.71	0.18	−0.94	−0.49
voicingvoiceless:vowel	−0.58	0.16	−0.78	−0.38

**Fig. 10.** Expected nasalisation proportion by vowel and voicing, with 95% CrIs.

have commented the following code, we encourage readers to test each line sequentially and inspect the intermediate output to fully understand the process. We assume that readers are familiar enough with models with varying terms (aka random effects, mixed-effects, hierarchical, multilevel, nested models). What readers should note is that to obtain the expected predictions of nasalisation proportion for each speaker, the constant terms and the varying terms should be added (since the varying terms indicate the deviation of each speaker from the overall estimate).

```
library(tidybayes)
```

```
nas_prop_r <- nas_prop_bm |>
# extract draws from model, only `r_speaker` varying terms
spread_draws(r_speaker[speaker,voicingvowel]) |>
# separate the column voicingvowel to two columns
separate(voicingvowel, c("voicing", "vowel")) |>
# join the draws with the `b_` terms
left_join(y = nas_prop_bm_draws_long) |>
```

Table 11First ten rows of posterior draws from *nas_prop_bm* in long format.

.chain	.iteration	.draw	type	voicing	vowel	value
1	1	1	b	voicingvoiced	vowela	−0.4632870
1	1	1	b	voicingvoiceless	vowela	−0.0943699
1	1	1	b	voicingvoiced	vowe	−0.1546258
1	1	1	b	voicingvoiceless	vowe	−0.2080810
1	1	1	b	voicingvoiced	vowel	−0.4810383
1	1	1	b	voicingvoiceless	vowel	−0.0529377
1	1	1	b	voicingvoiced	vowel	−0.3096219
1	1	1	b	voicingvoiceless	vowel	0.3212456
1	1	1	b	voicingvoiced	vowel	−0.6198312
1	1	1	b	voicingvoiceless	vowel	−0.3680900

```
# get the expected log-odd value of each speaker, in each draw
# this is the sum of the `value` from the b_ terms and the value
from the
# r_speaker term.
mutate(r_speaker_value = value + r_speaker)|>
# group the data for summarise
group_by(.draw, speaker, voicing) |>
# get mean expected log odds by draw, speaker and voicing
(averaging across vowel)
summarise(r_speaker_value_mean = mean(r_speaker_value))
|>
# make the data wider: two columns, one for voiced and one for
voiceless
pivot_wider(names_from = voicing, values_from =
r_speaker_value_mean) |>
# finally, calculate the difference in expected log odds of
voiceless and voiced
mutate(voicing_diff = voicingvoiceless - voicingvoiced)
```

Fig. 12 plots the posterior distributions of the expected log-odd difference of coarticulatory nasalisation in voiceless vs voiced NC sequences (x-axis), for each speaker in the data (y-axis), as predicted by the model. The red solid vertical line indicates the constant (overall) expected log-odd difference based on the (constant) *b_* terms. The black dashed vertical line marks log-difference 0 (i.e., no difference in proportion of nasalisation between voiceless and voiced NC).

```
nas_prop_r |>
ggplot(aes(voicing_diff, reorder(speaker, voicing_diff))) +
stat_halfeye() +
geom_vline(xintercept = mean(nas_prop_bm_diff$voicing_diff),
colour = "red") +
geom_vline(xintercept = 0, linetype = "dashed") +
lims(x = c(-1, 1.5)) +
labs(
x = "Log-odds difference between voiceless vs voiced NC
sequences",
y = "Speaker"
)
```

There is a lot of uncertainty within and between speakers: while the distributions of most speakers are located in the positive range, some expected distributions (see last 5 speakers at the bottom of figure) do substantially span both negative and positive values. In other words, while most speakers are more likely to have a larger nasalisation proportion in voiceless NC sequences, a few might in fact have the opposite pattern. Even

Table 12
Mean nasalisation proportion by draw and voicing, averaged across vowels (first 10 rows).

.draw	voicing	value_mean	value_mean_prop
1	voicingvoiced	-0.4056808	0.3999482
1	voicingvoiceless	-0.0804466	0.4798992
2	voicingvoiced	-0.3905744	0.4035790
2	voicingvoiceless	-0.1227124	0.4693603
3	voicingvoiced	-0.4436082	0.3908816
3	voicingvoiceless	-0.1320190	0.4670431
4	voicingvoiced	-0.4385607	0.3920840
4	voicingvoiceless	-0.0981768	0.4754755
5	voicingvoiced	-0.3827134	0.4054726
5	voicingvoiceless	-0.2657250	0.4339569

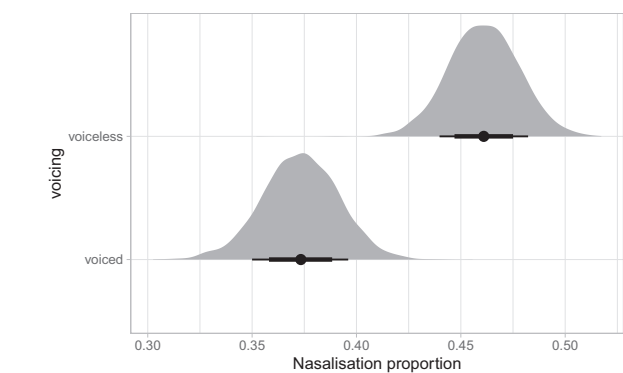


Fig. 11. Expected nasalisation proportion by voicing, averaged across vowels.

Table 13
Upper and lower limits of 90, 80 and 60 % Credible Intervals of the difference ratio of nasalisation proportion in vowels followed by voiced vs voiceless consonants.

q90	q80	q60
1.23	1.27	1.33
1.69	1.63	1.56

among those speakers that do have a more robust positive difference, there is a lot of uncertainty as to the magnitude of the difference.

To conclude, previous work on coarticulatory nasalisation in English has suggested that vowels followed by an NC sequence where C is voiceless (NT) should show earlier coarticulatory nasalisation than vowels followed by an NC sequence where C is voiced (ND). We fitted a Bayesian beta regression model to assess this expectation. In sum, we find an increase of nasalisation proportion in the voiceless NC sequences compared to that of voiced NC sequences, with a 90% probability that the increase is between 23% and 69% of the proportion of voiced NC sequences. As for individual speakers' variability, we find that while most speakers are more likely to have a larger nasalisation proportion in voiceless NC sequences, a few might have the opposite pattern.

6. Conclusion

In this tutorial we have presented two case studies where the outcome variable is a beta variable, i.e. a numeric continuous variable bounded between 0 and 1. We have shown how to fit Bayesian beta regressions to coarticulatory progressive

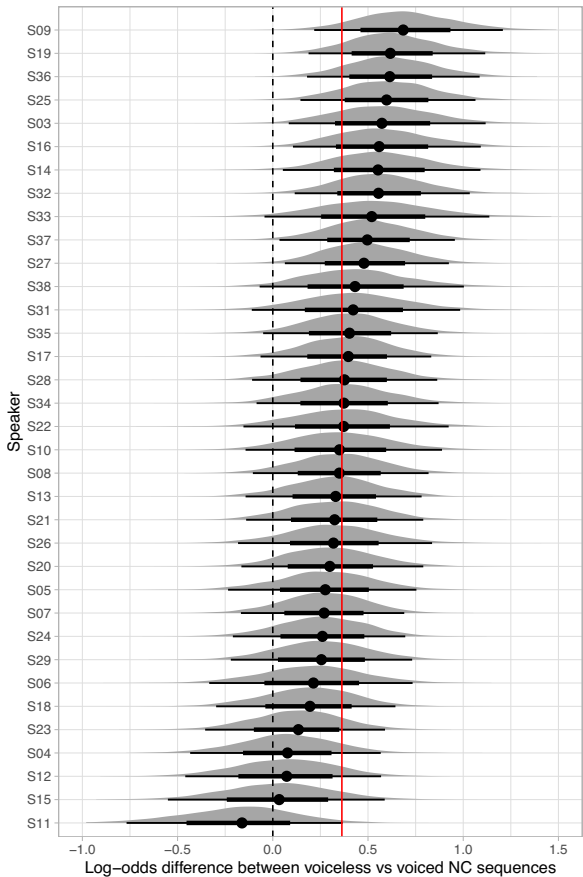


Fig. 12. Expected log-odd difference of coarticulatory nasalisation in voiceless vs voiced NC sequences for each speaker.

voicing in voiceless stops in Italian in Section 4 and coarticulatory regressive nasalisation in vowels in German in Section 5. While this tutorial alone will not be sufficient to be able to independently run full Bayesian analyses, it will serve as a first introduction for readers to jump start their learning.

CRedit authorship contribution statement

Paul-Christian Bürkner: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation. **Stefano Coretta:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Formal analysis, Data curation, Conceptualization.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- Abramson, A. S., & Whalen, D. H. (2017). Voice onset time (VOT) at 50: Teoretical and practical issues in measuring voicing distinctions. *Journal of Phonetics*, 63, 75–86. <https://doi.org/10.1016/j.wocn.2017.05.002>.
- Arel-Bundock, V., Greifer, N., & Heiss, A. (2024). How to interpret statistical models using marginaeffects for R and Python. *Journal of Statistical Software*, 111, 1–32. <https://doi.org/10.18637/jss.v111.i09>.
- Arel-Bundock, V., Greifer, N., & Heiss, A. (2025). How to interpret statistical models using marginaeffects in R and Python. *Journal of Statistical Software*.
- Baayen, R. H. (2008). *Analyzing linguistic data: A practical introduction to statistics using R*. Cambridge: Cambridge University Press. <https://doi.org/10.1017/CBO9780511801686>.
- Bates, D., Mächler, M., Bolker, B., & Walker, S. (2015). Fitting linear mixed-effects models using lme4. *Journal of Statistical Software*, 67, 1–48.
- Beddor, P. S. (2009). A coarticulatory path to sound change. *Language*, 785–821. <https://doi.org/10.1353/lan.0.0165>.
- Bockting, F., Radev, S.T., Bürkner, P.C., 2024. Simulation-based prior knowledge elicitation for parametric bayesian models. *Scientific Reports* 14, 17330. URL: <https://www.nature.com/articles/s41598-024-68090-7>, doi:10.1038/s41598-024-68090-7.
- Bonat, W.H., Jr, P.J.R., Zeviani, W.M., 2013. Regression models with responses on the unity Interval: Specification, estimation and comparison. *Biometric Brazilian Journal* 30, 18.
- Bürkner, P. C. (2017). brms: An R package for Bayesian multilevel models using Stan. *Journal of Statistical Software*, 80, 1–28. <https://doi.org/10.18637/jss.v080.i01>.
- Bürkner, P. C. (2018). Advanced Bayesian multilevel modeling with the R package brms. *The R Journal*, 10, 395–411. <https://doi.org/10.32614/RJ-2018-017>.
- Bürkner, P. C. (2021). Bayesian Item Response Modeling in R with brms and Stan. *Journal of Statistical Software*, 100. <https://doi.org/10.18637/jss.v100.i05>.
- Bürkner, P.C. (2025). The brms book: Applied Bayesian regression modelling using R and Stan.
- Bürkner, P.C., Gabry, J., Kay, M., Vehtari, A. (2024). posterior: Tools for working with posterior distributions.
- Carignan, C. (2021). A practical method of estimating the time-varying degree of vowel nasalization from acoustic features. *The Journal of the Acoustical Society of America*, 149, 911–922. <https://doi.org/10.1121/1.50002925>.
- Carignan, C., Shosted, R. K., Fu, M., Liang, Z. P., & Sutton, B. P. (2015). A real-time MRI investigation of the role of lingual and pharyngeal articulation in the production of the nasal vowel system of French. *Journal of Phonetics*, 50, 34–51. <https://doi.org/10.1016/j.wocn.2015.01.001>.
- Cho, T., Kim, D., & Kim, S. (2017). Prosodically-conditioned fine-tuning of coarticulatory vowel nasalization in English. *Journal of Phonetics*, 64, 71–89. <https://doi.org/10.1016/j.wocn.2016.12.003>.
- Coretta, S. (2019). Vowel duration, voicing duration, and vowel height: acoustic and articulatory data from Italian [research compendium]. doi:10.17605/OSF.IO/XDGFZ.
- Coretta, S. (2020). Vowel duration and consonant voicing: A production study. Ph.D. thesis. The University of Manchester.
- Coretta, S., Casillas, J. V., Roessig, S., Franke, M., Ahn, B., Al-Hoorie, A. H., Al-Tamimi, J., Alotaibi, N. E., AlShakhori, M. K., Altmiller, R. M., Arantes, P., Athanasopoulou, A., Baese-Berk, M. M., Bailey, G., Sangma, C. B. A., Beier, E. J., Benavides, G. M., Benker, N., BensonMeyer, E. P., Benway, N. R., Berry, G. M., Bing, L., Bjørndahl, C., Bolyanatz, M., Braver, A., Brown, V. A., Brown, A. M., Brugos, A., Buchanan, E. M., Butlin, T., Buxó-Lugo, A., Caillol, C., Cangemi, F., Carignan, C., Carraturo, S., Caudrelier, T., Chodroff, E., Cohn, M., Cronenberg, J., Crouzet, O., Dager, E. L., Dawson, C., Diantoro, C. A., Dokovova, M., Drake, S., Du, F., Dubuis, M., Duème, F., Durward, M., Egurtzegi, A., Elsherif, M. M., Esser, J., Ferragne, E., Ferreira, F., Fink, L. K., Finley, S., Foster, K., Foulkes, P., Franzke, R., Frazer-McKee, G., Fromont, R., García, C., Geller, J., Grasso, C. L., Greca, P., Grice, M., Grose-Hodge, M. S., Gully, A. J., Halfacre, C., Hauser, I., Hay, J., Haywood, R., Hellmuth, S., Hilger, A. I., Holliday, N., Hoogland, D., Huang, Y., Hughes, V., Icardo Isasa, A., Ichovska, Z. G., Jeon, H. S., Jones, J., Junges, M. N., Kaefer, S., Kaland, C., Kelley, M. C., Kelly, N. E., Kettig, T., Khattab, G., Koolen, R., Kraemer, E., Krajewska, D., Krug, A., Kumar, A., Lander, A., Lentz, G. O., Li, W., Li, Y., Lialiou, M., Lima, R. M., Lo, J. J. H., Lopez Otero, J. C., Mackay, B., MacLeod, B., Mallard, M., McDonnellogue, C. A. M., Moroz, G., Murali, M., Nalborczyk, L., Nenadic, F., Nieder, J., Nikolic, D., Nogueira, F. G. S., Offerman, H. M., Passoni, E., Péliissier, M., Perry, S. J., Pfiffner, A. M., Proctor, M., Rhodes, R., Rodríguez, N., Roepke, E., Röer, J. P., Sacco, L., Scarborough, R., Schaeffler, F., Schleef, E., Schmitz, D., Shiryayev, A., Sósokuthy, M., Spaniol, M., Stanley, J. A., Strickler, A., Tavano, A., Tomaschek, F., Tucker, B. V., Turnbull, R., Ugwuanyi, K. O., Urrestarazu-Porta, I., van de Vijver, R., Van Engen, K. J., van Miltenburg, E., Wang, B. X., Warner, N., Wehrle, S., Westerbeek, H., Wiener, S., Winters, S., Wong, S. G. J., Wood, A., Wottawa, J., Xu, C., Zárate-Sánchez, G., Zellou, G., Zhang, C., Zhu, J., & Roettger, T. B. (2023). Multidimensional signals and analytic flexibility: Estimating degrees of freedom in human-speech analyses. *Advances in Methods and Practices in Psychological Science*, 6. <https://doi.org/10.1177/25152459231162567>.
- Cribari-Neto, F., & Zeileis, A. (2010). Beta Regression in R. *Journal of Statistical Software*, 34. <https://doi.org/10.18637/jss.v034.i02>.
- Cribari-Neto, F., & Zeileis, A. (2010). Beta Regression in R. *Journal of Statistical Software*, 34. <https://doi.org/10.18637/jss.v034.i02>.
- Davidson, L. (2016). Variability in the implementation of voicing in American English obstruents. *Journal of Phonetics*, 54, 35–50. <https://doi.org/10.1016/j.wocn.2015.09.003>.
- Espinheira, P. L., Ferrari, S. L., & Cribari-Neto, F. (2008). On beta regression residuals. *Journal of Applied Statistics*, 35, 407–419.
- Etz, A., Gronau, Q. F., Dablander, F., Edelsbrunner, P. A., & Baribault, B. (2018). How to become a Bayesian in eight easy steps: An annotated reading list. *Psychonomic Bulletin & Review*, 25, 219–234. <https://doi.org/10.3758/s13423-017-1317-5>.
- Ferrari, S., & Cribari-Neto, F. (2004). Beta Regression for Modelling Rates and Proportions. *Journal of Applied Statistics*, 31, 799–815. <https://doi.org/10.1080/0266476042000214501>.
- Fowler, C. A. (2005). Parsing coarticulated speech in perception: Effects of coarticulation resistance. *Journal of Phonetics*, 33, 199–213. <https://doi.org/10.1016/j.wocn.2004.10.003>.
- Gelman, A. (2005). Analysis of variance—why it is more important than ever. *The Annals of Statistics*, 33. <https://doi.org/10.1214/009053604000001048>.
- Gelman, A., Hill, J., & Yajima, M. (2012). Why we (usually) don't have to worry about multiple comparisons. *Journal of Research on Educational Effectiveness*, 5, 189–211. <https://doi.org/10.1080/19345747.2011.618213>. URL: <http://www.tandfonline.com/doi/abs/10.1080/19345747.2011.618213>.
- Gick, B., Wilson, I., & Derrick, D. (2013). *Articulatory phonetics*. Wiley-Blackwell.
- Gupta, A. K., & Nadarajah, S. (2014). *Handbook of Beta Distribution and Its Applications*. Boca Raton: CRC Press. <https://doi.org/10.1201/9781482276596>.
- Heiss, A. (2021). A guide to modeling proportions with bayesian beta and zero-inflated beta regression models. URL: <https://www.andrewheiss.com/blog/2021/11/08/beta-regression-guide>. dOI: 10.59350/7p1a4-0tw75.
- Herbst, C. T., Schutte, H. K., Bowling, D. L., & Svec, J. G. (2017). Comparing chalk with cheese: The EGG contact quotient is only a limited surrogate of the closed quotient. *Journal of Voice*, 31, 401–409. <https://doi.org/10.1016/j.jvoice.2016.11.007>.
- Johnson, K. (2003). *Acoustic and auditory phonetics*. Wiley-Blackwell. <https://doi.org/10.1159/000078663>.
- Kay, M. (2019). *tidybayes: Tidy data and geoms for Bayesian models*.
- Kay, M. (2023). ggdist: Visualizations of distributions and uncertainty in the grammar of graphics. *IEEE Transactions on Visualization and Computer Graphics*, 1–11. <https://doi.org/10.1109/TVCG.2023.3327195>.
- Kieschnick, R., & McCullough, B. D. (2003). Regression analysis of variates observed on (0, 1): Percentages, proportions and fractions. *Statistical Modelling*, 3, 193–213. <https://doi.org/10.1191/1471082X03st053oa>.
- Kirby, J., & Sonderegger, M. (2018). Mixed-effects design analysis for experimental phonetics. *Journal of Phonetics*, 70, 70–85. <https://doi.org/10.1016/j.wocn.2018.05.005>.
- Kluender, K. R., Diehl, R. L., & Wright, B. A. (1988). Vowel-length differences before voiced and voiceless consonants: An auditory explanation. *Journal of Phonetics*, 16, 153–169. [https://doi.org/10.1016/S0095-4470\(19\)30480-2](https://doi.org/10.1016/S0095-4470(19)30480-2).
- Kosmidis, I., Zeileis, A., 2024. Extended-support beta regression for [0, 1] responses. doi:10.48550/ARXIV.2409.07233.
- Krysicki, W. (1999). On some new properties of the beta distribution. *Statistics & Probability Letters*, 42, 131–137. [https://doi.org/10.1016/S0167-7152\(98\)00197-7](https://doi.org/10.1016/S0167-7152(98)00197-7).
- Ladefoged, P., & Johnson, K. (2014). *A course in phonetics*. Cengage learning.
- López, F. O. (2013). A bayesian approach to parameter estimation in simplex regression model: A comparison with beta regression. *revista Colombiana de Estadística*, 21.
- McElreath, R. (2019). *Statistical rethinking: A Bayesian course with examples in R and Stan*. Boca Raton, FL: CRC Press.
- McElreath, R. (2020). *Statistical rethinking: A Bayesian course with examples in R and Stan*. Chapman & Hall/CRC texts in statistical science series. second edition ed., CRC Press, Boca Raton. OCLC: on1130764237.
- Mikkola, P., Martin, O. A., Chandramouli, S., Hartmann, M., Abril Pla, O., Thomas, O., Pesonen, H., Corander, J., Vehtari, A., Kaski, S., Bürkner, P. C., & Klami, A. (2024). Prior knowledge elicitation: The past, present, and future. *Bayesian Analysis*, 19. <https://doi.org/10.1214/23-BA1381>. URL: <https://projecteuclid.org/journals/bayesian-analysis/volume-19/issue-4/Prior-Knowledge-Elicitation-The-Past-Present-and-Future/10.1214/23-BA1381.full>.
- Nalborczyk, L., Batailler, C., Loevenbruck, H., Vilain, A., & Bürkner, P. C. (2019). An introduction to bayesian multilevel models using brms: A case study of gender effects on vowel variability in standard Indonesian. *Journal of Speech, Language, and Hearing Research*. https://doi.org/10.1044/2018_JSLHR-S-18-0006.
- Ohala, J. J. (2011). Accommodation to the aerodynamic voicing constraint and its phonological relevance, in. *In Proceedings of the 17th International Congress of Phonetic Sciences* (pp. 64–67).
- Politzer-Ahles, S., & Piccinini, P. (2018). On visualizing phonetic data from repeated measures experiments with multiple random effects. *Journal of Phonetics*. <https://doi.org/10.1016/j.wocn.2018.05.002>.
- Roettger, T. B., Winter, B., & Baayen, H. (2018). Emergent data analysis in phonetic sciences: Towards pluralism and reproducibility. *Journal of Phonetics*, 73, 1–7. <https://doi.org/10.1016/j.wocn.2018.12.001>.
- Rosen, K. M. (2005). Analysis of speech segment duration with the lognormal distribution: A basis for unification and comparison. *Journal of Phonetics*, 33, 411–426. <https://doi.org/10.1016/j.wocn.2005.02.001>.
- Rothenberg, M. (1967). *The breath-stream dynamics of simple-released-plosive production* (volume 6) Basel: Biblioteca Phonetica.

- Scholz, M., Bürkner, P.C. (2023). Posterior accuracy and calibration under misspecification in bayesian generalized linear models. arXiv preprint arXiv:2311.09081.
- Scholz, M., & Bürkner, P. C. (2025). Prediction can be safely used as a proxy for explanation in causally consistent bayesian generalized linear models. *Journal of Statistical Computation and Simulation*, 1–24.
- Stan Development Team (2017). Stan: A C++ library for probability and sampling, version 2.14.0.
- Stan Development Team, 2025. Prior Choice Recommendations. <https://github.com/stan-dev/stan/wiki/Prior-Choice-Recommendations>.
- Tamminga, M., Ahern, C., & Ecay, A. (2016). Generalized additive mixed models for intraspeaker variation. *Linguistics Vanguard*, 2, 33–41. <https://doi.org/10.1515/lingvan-2016-0030>.
- Tavakoli, S., Matteo, B., Pigoli, D., Chodroff, E., Coleman, J., Gubian, M., Renwick, M. E., & Sonderegger, M. (2024). Statistics in phonetics. *Annual Review of Statistics and Its Application*. <https://doi.org/10.1146/annurev-statistics-112723-034642>.
- van den Berg, J. (1958). Myoelastic-aerodynamic theory of voice production. *Journal of Speech and Hearing Research*, 1, 227–244. <https://doi.org/10.1044/jshr.0103.227>.
- Vasishth, S., Beckman, M., Nicenboim, B., Li, F., & Kong, E. J. (2018). Bayesian data analysis in the phonetic sciences: A tutorial introduction. *Journal of Phonetics*, 71, 147–161. <https://doi.org/10.1016/j.wocn.2018.07.008>.
- Verissimo, J. (2021). Analysis of rating scales: A pervasive problem in bilingualism research and a solution with Bayesian ordinal models. *Bilingualism: Language and Cognition*, 24, 842–848. <https://doi.org/10.1017/S1366728921000316>.
- Wickham, H., Averick, M., Bryan, J., Chang, W., McGowan, L., François, R., Golemund, G., Hayes, A., Henry, L., Hester, J., Kuhn, M., Pedersen, T., Miller, E., Bache, S., Müller, K., Ooms, J., Robinson, D., Seidel, D., Spinu, V., Takahashi, K., Vaughan, D., Wilke, C., Woo, K., & Yutani, H. (2019). Welcome to the Tidyverse. *Journal of Open Source Software*, 4, 1686. <https://doi.org/10.21105/joss.01686>.
- Winter, B. (2020). *Statistics for linguists: An introduction using R*. Routledge.