

Bayesian Distributional Structural Equation Modeling

in Stan and brms

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What is a latent variable?

Intelligence, creativity, personality traits, ...

What aspects of a latent variable may vary?

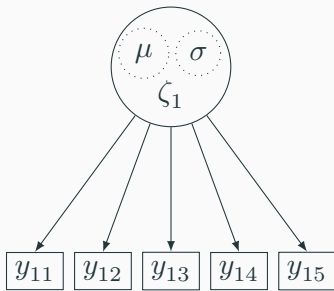
Reference: Fazio, L. & Bürkner, P. C. (2025). Gaussian distributional structural equation models: A framework for modeling latent heteroscedasticity. *Multivariate Behavioral Research*. doi:10.1080/00273171.2025.2483252

Motivational example: Predicting negative affect with neuroticism

- Neuroticism is a major personality trait describing a person's tendency to feel negative emotions such as anxiety, worry, or sadness
- Negative affect describes a person's current negative emotional state
- Neuroticism may not only predict the average negative affect but also its variability

Reference: Mader, N., Arslan, R. C., Schmukle, S. C., & Rohrer, J. M. (2023). Emotional (in)stability: Neuroticism is associated with increased variability in negative emotion after all. *Proceedings of the National Academy of Sciences of the United States of America*. doi:10.1073/pnas.2212154120

Latent variables with explicit distributional parameters

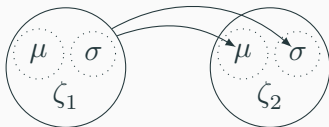


$$\zeta_1 \sim \text{normal}(\mu_1 = 0, \sigma_1)$$

$$y_{1i} \sim \text{normal}(\alpha_{1i} + \lambda_{1i} \zeta_1, \tau_{1i})$$

```
bformula <- bf(zeta1 | mi() ~ 0) + bf(y1i ~ mi(zeta1))
```

A simple distributional structural equation model

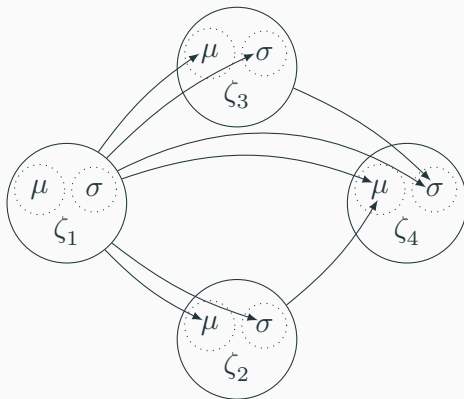


$$\mu_2 = \beta_{\mu_2 1} \zeta_1$$

$$\sigma_2 = \exp(\beta_{\sigma_2 0} + \beta_{\sigma_2 1} \zeta_1)$$

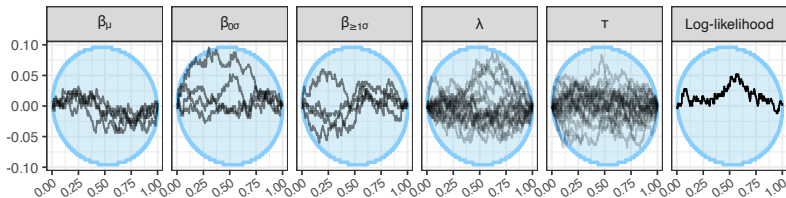
```
bformula <- bf(zeta1 | mi() ~ 0) +  
  bf(zeta2 | mi() ~ 0 + mi(zeta1), sigma ~ mi(zeta1))  
# + <measurement model formulas>
```

Advanced model: Mediation

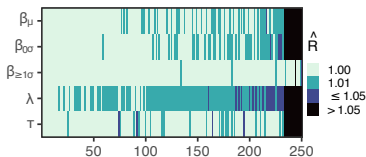


Results: Mediation

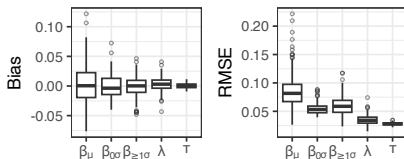
(a) Calibration



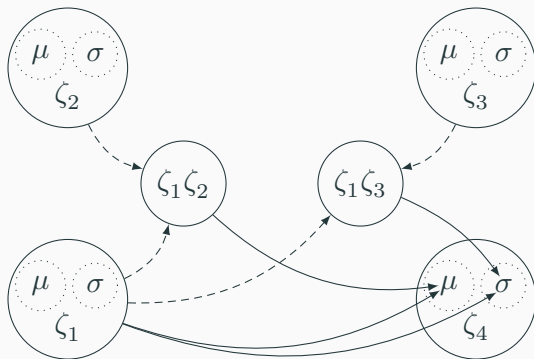
(b) Convergence



(c) Parameter recovery

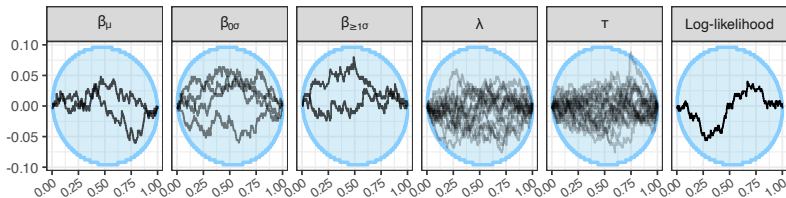


Advance model: Interaction

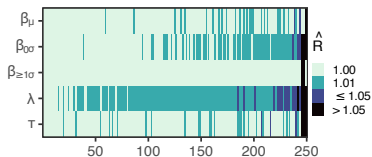


Results: Interaction

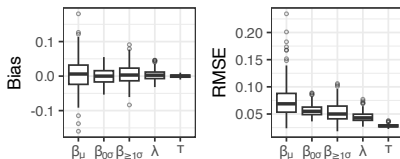
(a) Calibration



(b) Convergence



(c) Parameter recovery



What is a latent variable (again)?

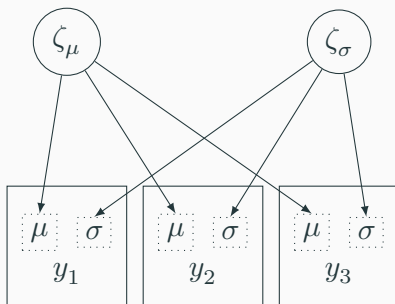
Reference: Fazio, L., Bürkner, P. C. (in review). Latent Variable Models for Distributional Features. *ArXiv preprint*. <https://arxiv.org/abs/2606.15526>

Motivational example: Predicting neuroticism with negative affect

- Not only the mean of the negative affect but also its variability may predict neuroticism
- (Essentially the previous example in reverse)

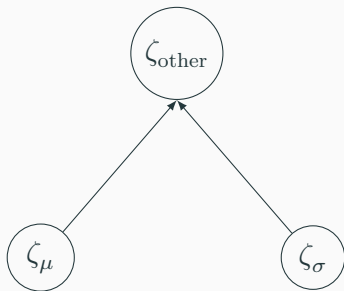
Reference: Mader, N., Arslan, R. C., Schmukle, S. C., & Rohrer, J. M. (2023). Emotional (in)stability: Neuroticism is associated with increased variability in negative emotion after all. *Proceedings of the National Academy of Sciences of the United States of America*. doi:10.1073/pnas.2212154120

A simple distributional measurement model



```
bformula <- bf(zeta_mu | mi() ~ 0) +  
  bf(zeta_sigma | mi() ~ 0) +  
  bf(yi ~ mi(zeta_mu), sigma ~ mi(zeta_sigma))
```

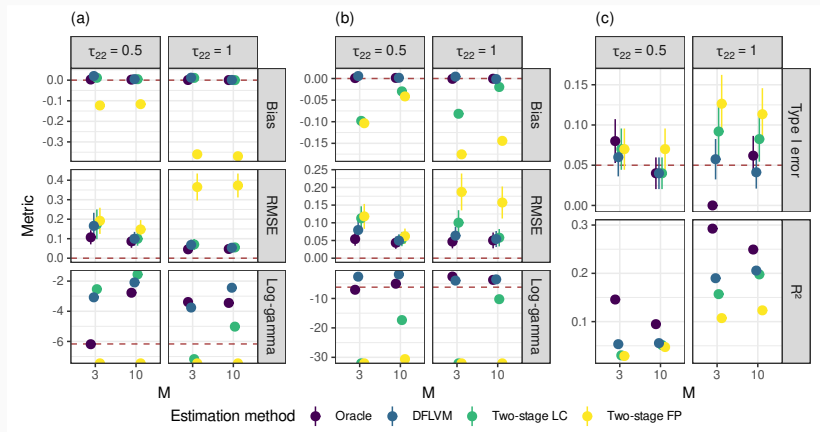
A simple LVM with distributional features



```
bformula <- bf(zeta_other ~ mi(zeta_mu) + mi(zeta_sigma))  
# + <measurement model formulas>
```

Results: Gaussian-identity model for ζ_{other}

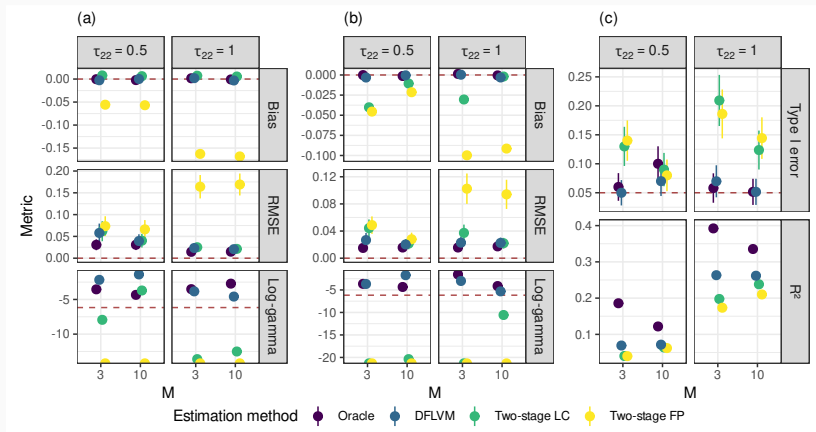
Results for the coefficient of the relation $\zeta_{\text{other}} \leftarrow \zeta_{\sigma}$



(a) unstandardized coefficient, (b) standardized coefficient, (c) scale independent metrics.
 τ_{22} = true standard deviation of ζ_{σ} .

Results: Poisson-log model for ζ_{other}

Results for the coefficient of the relation $\zeta_{\text{other}} \leftarrow \zeta_{\sigma}$



(a) unstandardized coefficient, (b) standardized coefficient, (c) scale independent metrics.

τ_{22} = true standard deviation of ζ_{σ} .

Distributional LVMs have a lot of potential for psychological research

Stan and brms do a good but imperfect job at estimating their posteriors

Challenges:

- Simulating realistic data from distributional LVMs is non-trivial
- Distributional LVMs pose problems even for Stan's sampler
- Convergence issues are hard to completely eliminate
- Model parameterization and identifiability constraints likely crucial